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On one problem of reflection of a Gaussion envelope electromagnetic pulse from a moving inhomogeneous medium

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The problem of plane electromagnetic wave propagation in moving isotropic/anisotropic, homogeneous media has been studied by many workers (Kong & Chang 1968, Okhubo 1971, Tai 1964); using the covariance of Maxwell's equations and the principle of phase invariance and many interesting results, including modifications of Snell's and Fresnel's laws have been obtained. However, to date, no such attempt has been made for the case of a moving inhomogeneous medium; though few studies have been made using other approaches (Kong 1971, Lee & Papas 1963, Tischer 1960). The reason for this is, that the Maxwell's equations do not admit plane wave solutions in the moving inhomogeneous media and hence, preclude the possibility of devising an invariant phase in the absence of which the principle of phase invariance cannot be applied. But in the studies of plane wave propagation in stationary inhomogeneous media, extensive use has been made of WKB method (Brekhovskikh 1960, Wait 1960), and under a certain condition (Brekhovskikh 1960, Mott 1958) the WKB solutions are exact solutions of the wave equation. The expression for the refractive index profile of an inhomogeneous medium in which the above condition is satisfied can be obtained (Burman 1966), hence for this profile, a reflected wave that is completely uncoupled from the incident wave can be defined i.e. the Maxwell's equations admit plane wave solutions which means that the concept of invariant phase can be applied.

In this communication we have investigated the nature of the reflected pulse from a moving inhomogeneous medium, which is characteristised by the above mentioned refractive index profile, when the incident pulse is an AM Gaussian envelope pulse.

The refractive index profile of the moving inhomogeneous medium under consideration (in the rest frame $K'(x', y', z')$ of the moving medium), is given by :

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$$n(z') = (1+z'/a')^{-2}, \qquad z' > 0$$
$$= 1 \qquad z' < 0 \qquad \ldots (1)$$

where a', is the inhomogeneity parameter independent of z', the medium is moving with a uniform velocity v, in z-direction (with respect to $K(x, y, z)$, the rest frame of the observer and is coincident with K' at $t' = 0$; x-z-being the plane of incidence). At $z' = 0$, the refractive index defined by eq. (1), is continuous but its gradient is discontinuous (for details see, Burman 1966). Let a plane wave be normally incident from the negative z'-direction, then the amplitude reflection co-efficient R', referred to gradient discontinuity at $z' = 0$ is (Burman 1966) :

$$R' = -1/[1+iza'k'] \qquad \ldots (2)$$

where $k' = \omega'/c$ ($\omega' = 2\pi f$, is the angular frequency of the wave in k', and c, is the velocity of light in vacuum). Now, making use of covariance of Maxwell's equations and phase invariance principle, R', in K, transforms (Yeh 1965) to :

$$R = -\left(\frac{1-\beta}{1+\beta}\right) \Big/ \left[1+i\ \frac{2a'}{c}\ \frac{\omega}{\gamma(1+\beta)}\right] \qquad \ldots (3)$$

where $v = \beta c$ and $\gamma^{-2} = 1-\beta^2$.

Let the incident source pulse be given by :

$$E_i(t_i) = \exp(-\sigma^2 t_i^2)\mathrm{Cos}(\omega_0 t_i) \qquad \ldots (4)$$

whose frequency spectrum is :

$$E_i(\omega) = \frac{\sqrt{\pi}}{2\sigma}\,[e^{-(\omega-\omega_0)2/4\sigma^2}+e^{-(\omega+\omega_0)/4\sigma^2}] \qquad \ldots (5)$$

where ω_0 is the carrier frequency, σ is related to the width of the pulse and $t_i = (z/c-t)$.

If the transfer function of the free space-moving inhomogeneous medium interface is of the form

$$|R(\omega)|\exp[-i\phi(\omega)]$$

then the reflected pulse in the region $z < 0$ is given by

$$E_r(t_r) = \frac{1}{2\pi}\int_{-\infty}^{\infty} |R(\omega)|\,E_i(\omega)\exp\{-i[\phi(\omega)-\omega t_r]\}d\omega \qquad \ldots (6)$$

where $t_r = -(z/c-t)$.

Substituting eq. (5), in eq. (6), and then evaluating the integral approximately by stationary phase method (for details see, Ginzburg 1970); the reflected pulse is given as :

$$E_r(t_r) = |R(^0\omega)| \frac{e^{-[\sigma(t_r-\phi'(\omega_0))]^2} \Big/ \left[1+\left\{\frac{\phi''(\omega_0)}{2}(2\sigma)^2\right\}^2\right]}{\left[1+\left\{\frac{\phi''(\omega_0)}{2}(2\sigma)^2\right\}\right]^{1/4}}$$

$$\times \cos\left[\omega_0 t_r - \phi(\omega_0) - \tfrac{1}{2}\tan^{-1}\left\{\frac{\phi''(\omega_0)}{2}(2\sigma)^2\right\}\right.$$

$$\left. + \frac{[\sigma(t_r-\phi'(\omega_0))]^2}{\left[1+\left\{\frac{\phi''(\omega_0)}{2}(2\sigma)^2\right\}^2\right]} \frac{\phi''(\omega_0)}{2}(2\sigma)^2 \pm \pi/2\right] \tag{7}$$

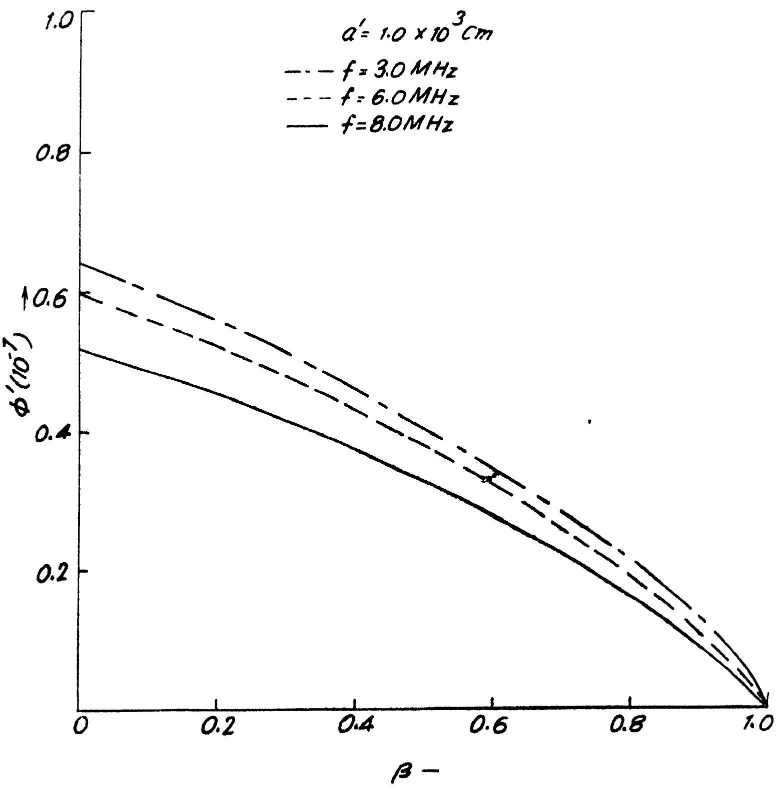

FIG. 1

Fig. 1. Variation of $\phi'(\omega_0)$ as a function of velocity parameter β, for various values of carrier frequency f_0.

where

$$|R(\omega_0)| = |R(\omega)|_{\omega=\omega_0} = \left(\frac{1-\beta}{1+\beta}\right)\left[1+\left\{\frac{2a'\omega_0}{\gamma c(1+\beta)}\right\}^2\right]^{-\frac{1}{2}}$$

$$\phi(\omega_0) = \phi(\omega)|_{\omega=\omega^0} = \tan^{-1}\left[1+\left\{\frac{2a'\omega_0}{\gamma c(1+\beta)}\right\}^2\right]^{-1}$$

$$\phi'(\omega_0) = \frac{d\phi(\omega)}{d\omega}\bigg|_{\omega=\omega_0} \quad \frac{2a'}{\gamma c(1+\beta)}\left[1+\left\{\frac{2a'\omega_0}{\gamma c(1+\beta)}\right\}^2\right]^{-1}$$

$$\phi''(\omega_0) = \frac{d^2\phi(\omega)}{d\omega^2}\bigg|_{\omega=\omega_0} = z\left[\frac{2a'}{\gamma c(1+\beta)}\right]^3\left[1+\left\{\frac{2a'\omega_0}{\gamma c(1+\beta)}\right\}^2\right]^{-2} \quad \ldots (8)$$

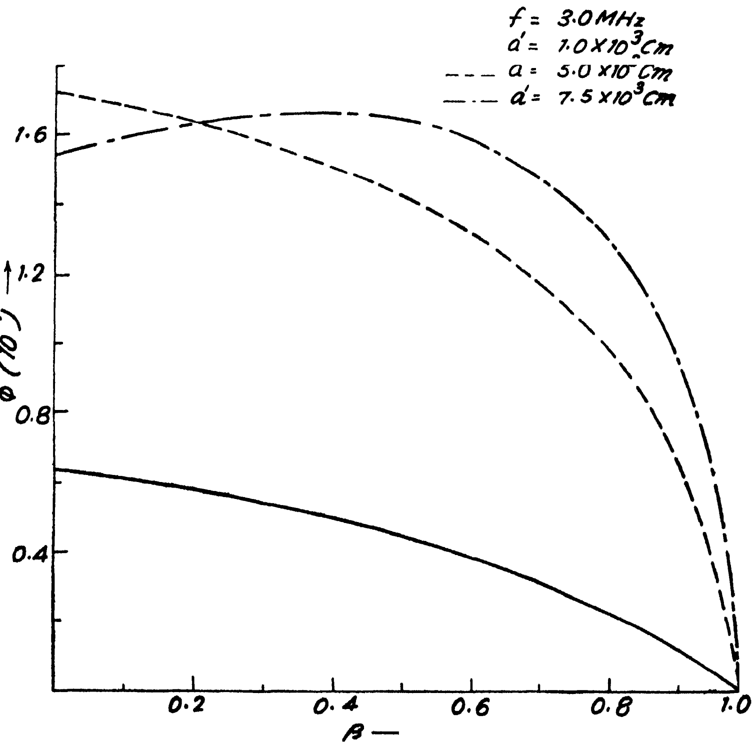

FIG. 2

Fig. 2. Variation of $\phi'(\omega_0)$ as a function of velocity parameter β, for various values of inhomogeneity parameter a'.

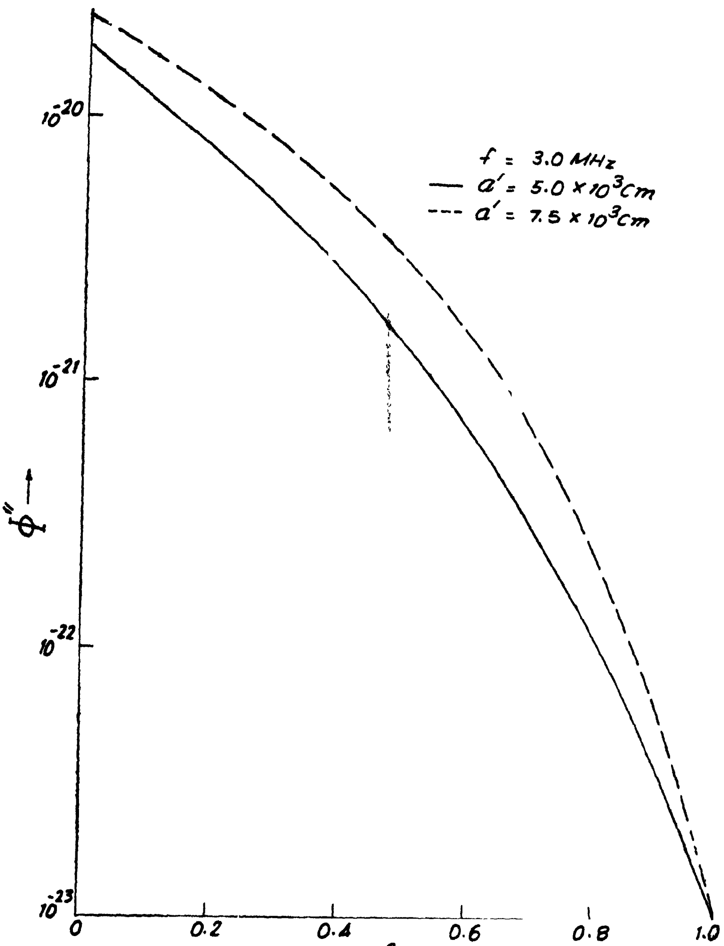

FIG.3

Fig. 3. Variation of $\phi''(\omega_0)$ as a functiou of velocity parameter β, for various values of -inhomogeneity paramoter a'.

From eq. (7), it is evident that reflected pulse is shifted on time axis, and the whole pulse is delayed by the group delay time τ_g, defined as,

$$\tau_g = \phi'(\omega_0).$$

Let us define a parameter l_0, as

$$l_0 = \phi''(\omega_0)(2\sigma)^2/2$$

which determines the distortion of the pulse, after it has undergone reflection. In case $l_0 \gg 1$, the reflected pulse is no longer Gaussian due to heavy distortion, but in case $l_0 \ll 1$, the reflected pulse retains its envelope. Eq. (8), shows that both b , and l_0, are complicated functions of the velocity of the moving inhomogeneous medium and the reflected pulse is Doppler-shifted.

Some numerical results have been presented in Figures (1), (2) and (3), which depict the dependence of $\phi''(\omega_0)$ and $\phi s(\omega_0)$, on the velocity of the medium v, inhomogeneity parameter a', and carrier frequency f_0, It is seen that higher the velocity of the medium, smaller is the delay and dostortion of the reflected pulse.

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Non-applicability of Wasastjerna potential function for heavy metal halide molecules

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A large number of potential energy functions have been used by different workers for the purpose of evaluating binding energies of diatomic molecules. Rittner (1951) suggested a potential function consisting of electrostatic, Van der Waals, polarization and an exponential type of overlap. However, due to its lengthy and complicated form, attempts have been made to find out some simpler form